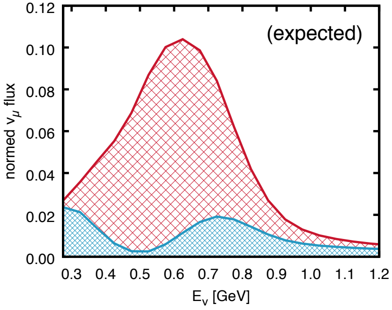
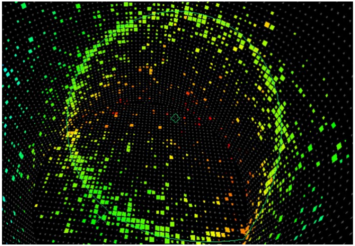
Resonance Pion Production in NuWro

Kajetan Niewczas

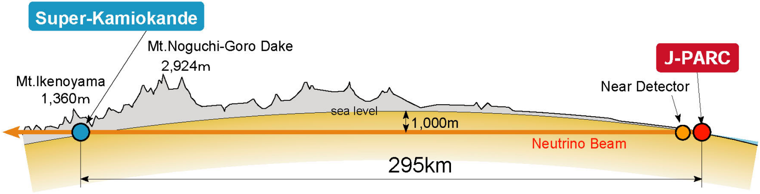
Neutrino oscillation experiments

$$P_{2\mathrm{f}}(\nu_\mu \to \nu_\mu) = 1 - \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2 L}{4E_\nu}\right)$$

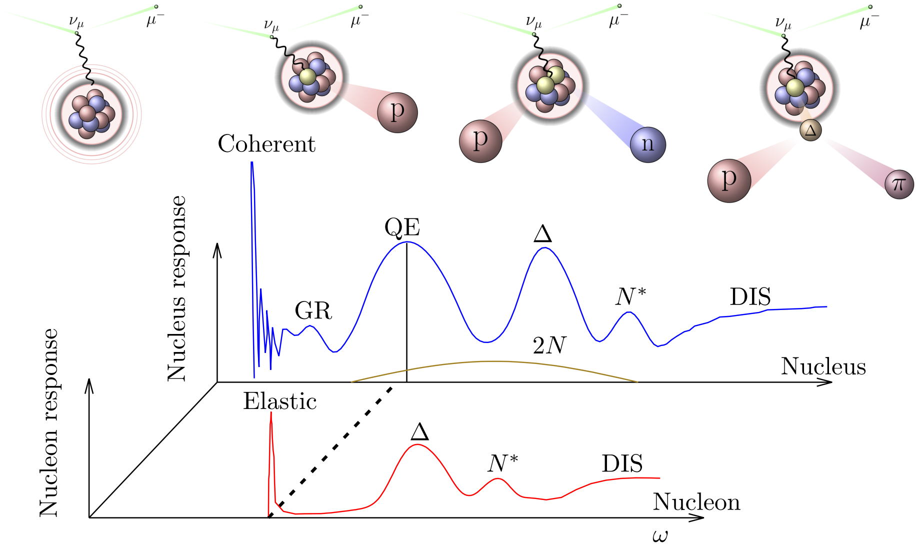




$$E_\nu^{\mathrm{rec}} = \frac{2(M_n - E_B)E_\mu - (E_B^2 - 2M_nE_B + m_\mu^2)}{2[M_n - E_B - E_\mu + |\vec{k}_\mu|\cos\theta_\mu]}$$

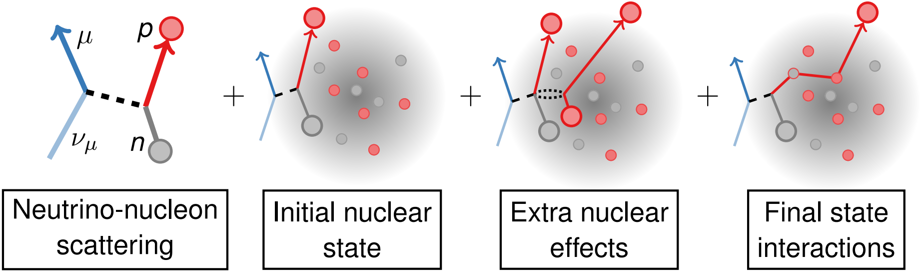


Nuclear response



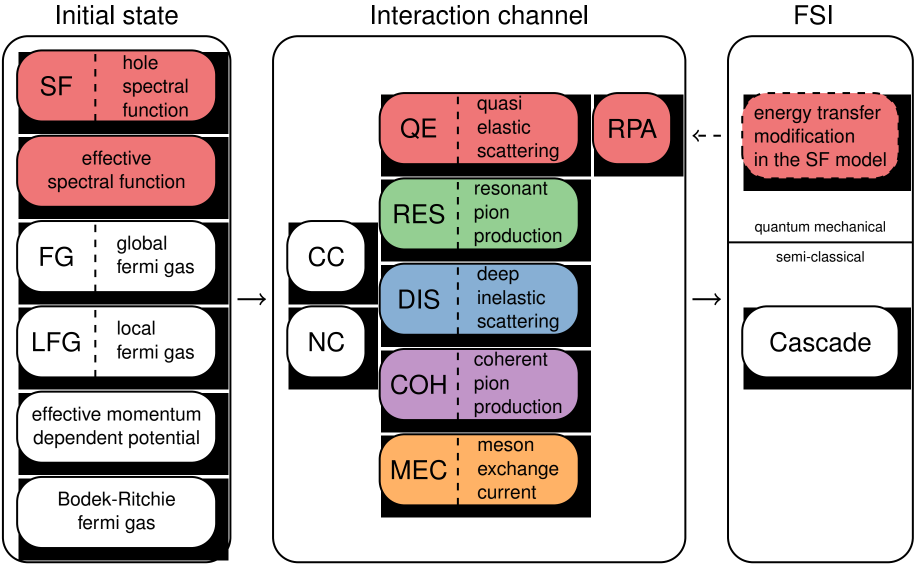
T. Van Cuyck

Cross section in the factorized scheme

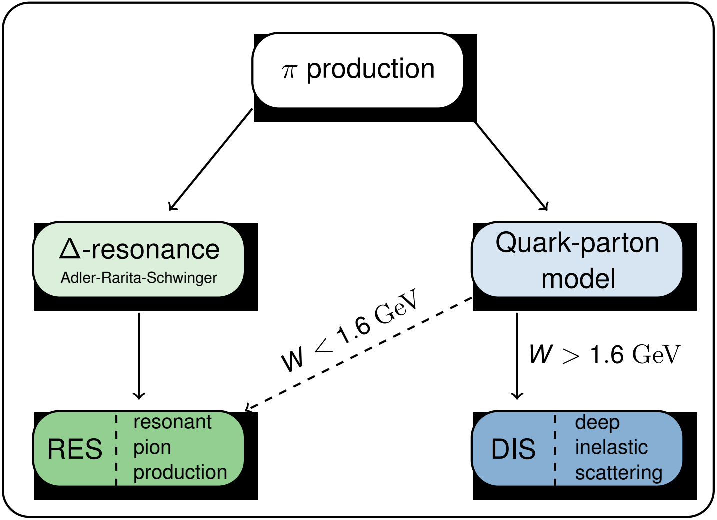


- **Neutrino-nucleon scattering**: elementary interaction cross section
- **Initial nuclear state**: modeling nucleons in the nuclear medium before the weak interaction
- **Extra nuclear effects**: multiple-nucleon interactions or correlations
- **Final state interactions**: in-medium outgoing particle propagation

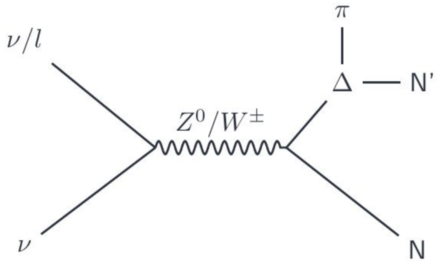
NuWro blueprint



Pion production in NuWro



Resonant pion production



The following channels are considered:

$$\nu + p \to l^- + (\Delta^{++} \to p + \pi^+) \qquad \bar{\nu} + n \to l^+ + (\Delta^- \to n + \pi^-)$$

$$\nu + n \to l^- + (\Delta^+ \to p + \pi^0 \text{ or } n + \pi^+) \quad \bar{\nu} + p \to l^+ + (\Delta^0 \to p + \pi^- \text{ or } n + \pi^0)$$

$$\nu(\bar{\nu}) + p \to \nu(\bar{\nu}) + (\Delta^+ \to p + \pi^0 \text{ or } n + \pi^+)$$

$$\nu(\bar{\nu}) + n \to \nu(\bar{\nu}) + (\Delta^0 \to p + \pi^- \text{ or } n + \pi^0)$$

Dimensionality of the problem

Δ-resonance excitation (free nucleon)	Pion production off a nucleon	Pion production on a nucleus
$\frac{\mathrm{d}^2\sigma}{\mathrm{d}Q^2\mathrm{d}W}$	$\frac{\mathrm{d}^4\sigma}{\mathrm{d}Q^2\mathrm{d}W\mathrm{d}\Omega_\pi^*}$	$\frac{\mathrm{d}^8\sigma}{\mathrm{d}Q^2\mathrm{d}W\mathrm{d}\Omega_\pi^*\mathrm{d}E_m\mathrm{d}\vec{p}_m}$

+1 invariant variable: the cross section is always symmetric w.r.t. 1 azimuthal angle, e.g., ϕ_μ

Adler-Rarita-Schwinger formalism

Double-differential cross section for the Δ **production**:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}W\mathrm{d}Q^2} = G^2\cos^2\theta_C\frac{Wg(W)}{\pi^2 ME_\nu^2}\left[-(Q^2+m^2)V_1 + \frac{V_2}{M^2}\left(2(pq)(pk')\frac{M^2}{2}(Q^2+m^2)\right)\right.$$
$$\left. -\frac{V_3}{M^2}\left(Q^2(kp)-\frac{1}{2}(Q^2+m^2)(pq)\right)+\frac{V_4}{m^2}\frac{m^2}{2}-2\frac{V_5}{M^2}m^2(kp)\right]$$

where V_i are structure functions made of **hadronic tensor elements** and

$$g(W) = \frac{\Gamma_\Delta/2}{(W-M_\Delta)^2+\Gamma_\Delta^2/4}$$

is the **Breit-Wigner** formula introducing the Δ **width** (Γ_Δ)

S. L. Adler, Annals Phys. 50 (1968) 189-311; S. L. Adler, Phys.Rev. D12 (1975) 2644

Rarita-Schwinger field Ψ_μ

→ The **final hadronic state** is a $\frac{3}{2}$-**spin resonance** described as a **Rarita-Schwinger field**

→ The **transition** from the **nucleon** to, e.g., Δ^{++} **state** is given as a matrix element of the **weak hadronic current**: $\mathcal{J}_\mu^{CC} = \mathcal{J}_\mu^V + \mathcal{J}_\mu^A$

$$\langle \Delta^{++}(p') | \mathcal{J}_\mu^V | N(p) \rangle = \sqrt{3}\bar{\Psi}_\lambda(p') \left[g_\mu^\lambda \left(\frac{C_3^V(Q^2)}{M}\gamma_\nu + \frac{C_4^V(Q^2)}{M^2}p'_\nu + \frac{C_5^V(Q^2)}{M^2}p_\nu \right) q^\nu - q^\lambda \left(\frac{C_3^V(Q^2)}{M}\gamma_\nu + \frac{C_4^V}{M^2}p'_\nu + \frac{C_5^V(Q^2)}{M^2} \right) \right] \gamma_5 u(p)$$

$$\langle \Delta^{++}(p') | \mathcal{J}_\mu^A | N(p) \rangle = \sqrt{3}\bar{\Psi}_\lambda(p') \left[g_\mu^\lambda \left(\gamma_\nu \frac{C_3^A(Q^2)}{M} + \frac{C_4^A(Q^2)}{M^2} \right) q^\nu - q^\lambda \left(\frac{C_3^A(Q^2)}{M}\gamma_\mu + \frac{C_4^A(Q^2)}{M^2}p'_\mu \right) + g_\mu^\lambda C_5^A(Q^2) + \frac{q^\lambda q_\mu}{M^2} C_6^A(Q^2) \right] u(p)$$

Hadronic tensor $W_{\mu\nu}$

Defined as

$$W_{\mu\nu} = \frac{1}{4MM_\Delta}\frac{1}{2}\sum_{\text{spin}} \left\langle \Delta^{++}(p') \right| \mathcal{J}_\mu^{CC} \left| N(p) \right\rangle \left\langle \Delta^{++}(p') \right| \mathcal{J}_\nu^{CC} \left| N(p) \right\rangle^* \times \frac{\Gamma_\Delta/2}{(W-M_\Delta)^2+\Gamma_\Delta^2/4}$$

$\Gamma_\Delta(W)$ is the Δ width, for which we assume the P-wave (l=1) expression

$$\Gamma_\Delta = \Gamma_0 \left(\frac{q_{cm}(W)}{q_{cm}(W_\Delta)}\right)^{2l+1} \frac{M_\Delta}{W}$$

with

$$q_{cm}(W) = \sqrt{\left(\frac{W^2+M^2-m_\pi^2}{2W}\right)^2 - M^2}$$

$\Gamma_0 = 120\ \mathrm{MeV}$, $M_\Delta = 1232\ \mathrm{MeV}$, $m_\pi = 139.57\ \mathrm{MeV}$

Form Factors

Elementary information lies in **vector** and **axial** **form factors** ${C^{V,A}}_i$

There are **several parametrizations available** in NuWro

Our default choice:

C_5^A axial form factor from bubble chamber experiments

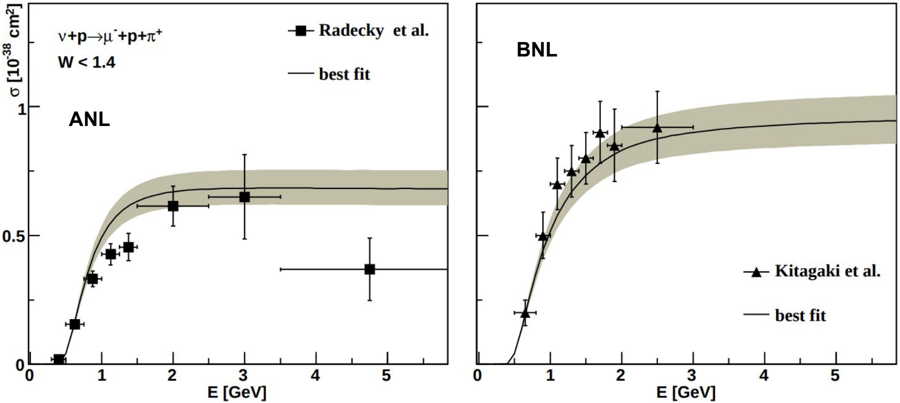
[K. M. Graczyk, D. Kielczewska, P. Przewlocki, and J. T. Sobczyk, Phys.Rev. D80 (2009) 093001]

- $\rightarrow$ A **consistent fit** to **both ANL** and **BNL** data
- $\rightarrow$ **Only** Δ^{++} channel **assuming** there is **no background**
- $\rightarrow$ **Consistency with NuWro**: only Δ^{++} in the given channel

Dipole parametrization, $M_A = 0.94\ \mathrm{GeV}$, $C_5^A(0) = 1.19$!

+ vector part from [O. Lalakulich, E. A. Paschos, G. Piranishvili, Phys.Rev. D 74 (2006) 014009]

Comparison with ANL/BNL data



→ **a simultaneus fit to ANL and BNL that shows their consistency !**

K. M. Graczyk, D. Kielczewska, P. Przewlocki, and J. T. Sobczyk, Phys.Rev. D80 (2009) 093001

Dimensionality of the problem

Δ-resonance excitation (free nucleon)	Pion production off a nucleon	Pion production on a nucleus
$\frac{\mathrm{d}^2\sigma}{\mathrm{d}Q^2\mathrm{d}W}$	$\frac{\mathrm{d}^4\sigma}{\mathrm{d}Q^2\mathrm{d}W\mathrm{d}\Omega^*_\pi}$	$\frac{\mathrm{d}^8\sigma}{\mathrm{d}Q^2\mathrm{d}W\mathrm{d}\Omega^*_\pi\mathrm{d}E_m\mathrm{d}\vec{p}_m}$

+2

Include angular information about the Δ decay (Ω^*_π)

+1 invariant variable: the cross section is always symmetric w.r.t. 1 azimuthal angle, e.g., ϕ_μ

Pion production off a nucleon

To produce an event, one needs **information about** the **produced pion**

Delta decays in the **hadronic CMS**:

$$\frac{\mathrm{d}^2\sigma_\Delta}{\mathrm{d}Q^2\mathrm{d}W} \to \frac{\mathrm{d}^4\sigma_\pi}{\mathrm{d}Q^2\mathrm{d}W\mathrm{d}\Omega_\pi^*} \times f_\Delta(\Omega_\pi^*)$$

Pion angular distributions are essential to **generate** the **kinematics**

In **NuWro**, it is taken from **experimental results** (ANL or BNL):

S.J. Barish et al., Phys.Rev. D19 (1979) 2511

G.M. Radecky et al., Phys.Rev. D25 (1982) 1161

T. Kitagaki et al., Phys.Rev. D34 (1986) 2554

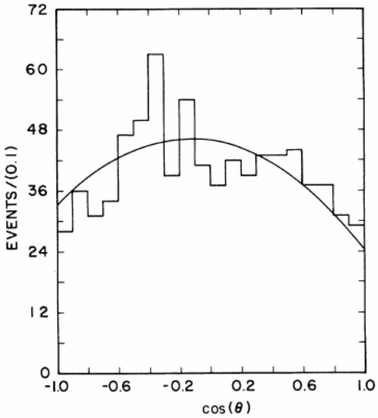
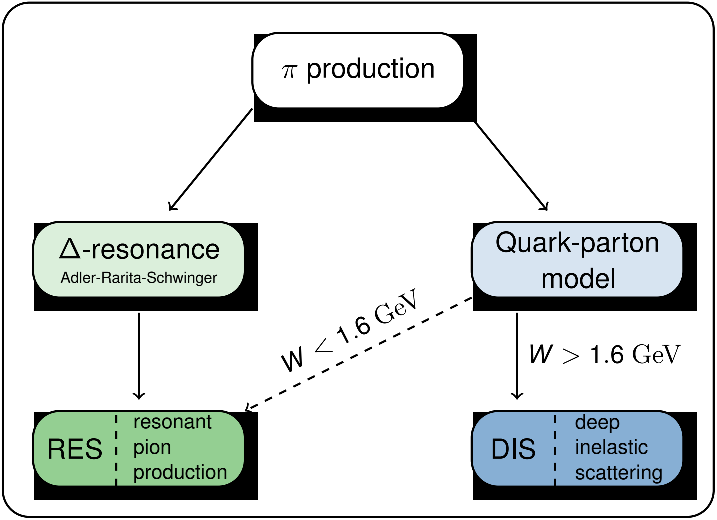


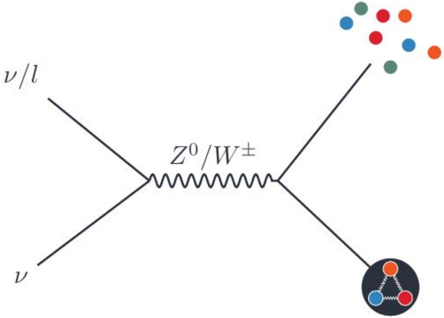
FIG. 15. Distribution of events in the pion polar angle $\cos\theta$ for the final state $\mu^- p \pi^+$, with $M(p\pi^+) < 1.4$ GeV. The curve is the area-normalized prediction of the Adler model.

Radecky et al. [ANL Collaboration], PRD 25 (1982) 1161

Pion production in NuWro



Deep inelastic scattering in NuWro



Events with invariant mass $W > 1.6\ \mathrm{GeV}$ are considered within the quark-parton model and labeled as DIS:

$$\nu + N \rightarrow l^- + X \qquad \bar{\nu} + N \rightarrow l^+ + X$$

$$\nu(\bar{\nu}) + N \rightarrow \nu(\bar{\nu}) + X$$

DIS cross section

Double-differential cross section expressed in terms of $x = Q^2/2M\omega$, $y = \omega/E_\nu$:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x\mathrm{d}y} = \frac{G^2 M E_\nu}{\pi(1+Q^2/M_{W,Z}^2)^2}\left[y\left(xy + \frac{m^2}{2E_\nu M}\right)F_1(x,Q^2) + \left(1 - y - \frac{Mxy}{2E_\nu} - \frac{m^2}{4E_\nu^2} - \frac{m^2}{2ME_\nu x}\right)F_2(x,Q^2) \pm \left(xy\left(1-\frac{y}{2}\right) - y\frac{m^2}{4ME_\nu}\right)F_3(x,Q^2)\right]$$

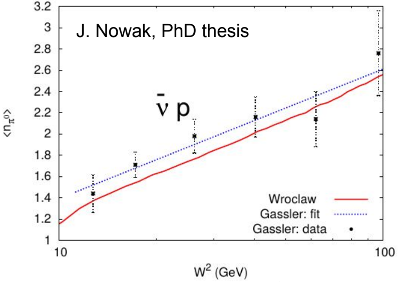
where $F_{1,2,3}$ are expressed by the **parton distribution functions**

$\rightarrow$ **GRV95 parametrization + low-Q^2 Bodek-Yang corrections**

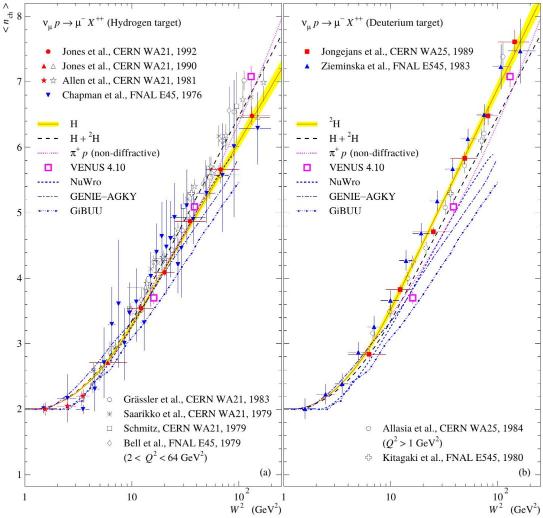
Hadronization

→ Hard-crafted **parameters tuned** to **experimental data**

→ Performed using **Pythia6** routines



Multiplicity of π^0



Multiplicity of π^+

Transition region & Non-resonant background

The **background extrapolated** from the **DIS** region (SPP + more)

Smooth SPP transition from **RES** to **DIS** in the W range $(1.3, 1.6)\ \mathrm{GeV}$:

$$\frac{\mathrm{d}\sigma^{\mathrm{SPP}}}{\mathrm{d}W} = \frac{\mathrm{d}\sigma^{\Delta}}{\mathrm{d}W}(1 - \alpha(W)) + \frac{\mathrm{d}\sigma^{\mathrm{DIS}}}{\mathrm{d}W} F^{\mathrm{SPP}} \alpha(W)$$

where $\alpha(W)$ assures a **smooth transition** and F^{SPP} is the **fraction** of **single pion production** in DIS

$$\begin{aligned}\alpha(W) = {} & \Theta(W_{min} - W)\frac{W - W_{th}}{W_{min} - W_{th}}\alpha_0 \\ & + \Theta(W_{max} - W)\Theta(W - W_{min})\frac{W - W_{min} + \alpha_0(W_{max} - W)}{W_{max} - W_{min}} \\ & + \Theta(W - W_{max})\end{aligned}$$

channel	$\nu_l\, p \to l^-\, p\, \pi^+$	$\nu_l\, n \to l^-\, n\, \pi^+$	$\nu_l\, n \to l^-\, p\, \pi^0$	$\bar{\nu}_l\, n \to l^+\, n\, \pi^-$	$\bar{\nu}_l\, p \to l^+\, p\, \pi^-$	$\bar{\nu}_l\, p \to l^+\, n\, \pi^0$
α_0	0.0	0.2	0.3	0.0	0.2	0.3

For all NC SPP channels: $\alpha_0 = 0$

Dimensionality of the problem

Δ-resonance excitation (free nucleon)	Pion production off a nucleon	Pion production on a nucleus
$\frac{\mathrm{d}^2\sigma}{\mathrm{d}Q^2\mathrm{d}W}$	$\frac{\mathrm{d}^4\sigma}{\mathrm{d}Q^2\mathrm{d}W\mathrm{d}\Omega_\pi^*}$	$\frac{\mathrm{d}^8\sigma}{\mathrm{d}Q^2\mathrm{d}W\mathrm{d}\Omega_\pi^*\mathrm{d}E_m\mathrm{d}\vec{p}_m}$

+4

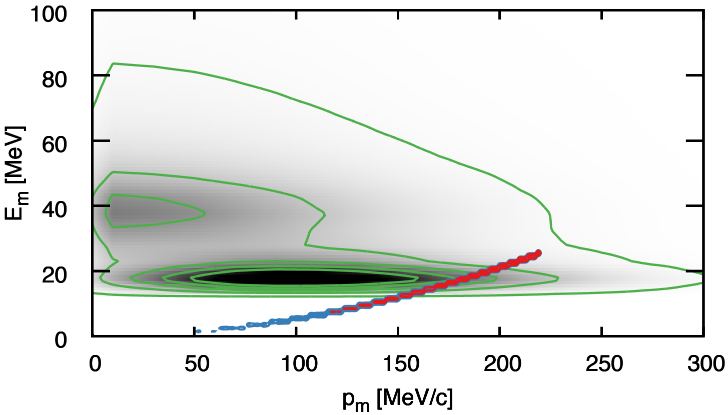
Put the target nucleon inside a nucleus ($E_m, \vec{p}_m$)

+1 invariant variable: the cross section is always symmetric w.r.t. 1 azimuthal angle, e.g., ϕ_μ

Pion production on a nucleus

Nucleon energy (E_m) and momentum ($|\vec{p}_m|$) are taken from the **available nuclear models** (Fermi motion is isotropic):

$\rightarrow$ Fermi gas, local Fermi gas, effective Spectral Function, ...

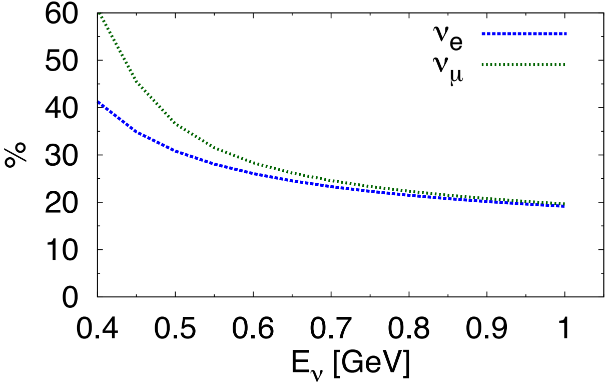


On-shell cross section and **off-shell kinematics** for the Δ excitation

$\Delta(1232)$ self-energy effects

- **In-medium** modification of the Δ-width using **Oset, Salcedo model**
  [E. Oset, L.L. Salcedo, Nucl. Phys. A468 (1987) 631]

- Approximated fraction of "pion-less Δ decays"
  [J. Sobczyk, J. Żmuda, Phys.Rev. C87 (2013) 065503]



Intranuclear cascade

- **Propagates particles** through the nuclear medium
- **Probability** of passing a distance λ:

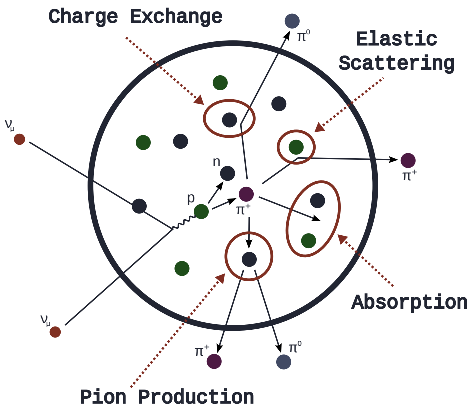
$$P(\lambda) = e^{-\lambda/\tilde{\lambda}}$$

where $\tilde{\lambda} \equiv (\rho\sigma)^{-1}$
ρ - local density
σ - cross section

- Implemented for **nucleons** and **pions**

T. Golan, C. Juszczak, J.T. Sobczyk, Phys.Rev. C86 (2012) 015505

- **Semi-classical** – neglects quantum mechanical effects



T. Golan

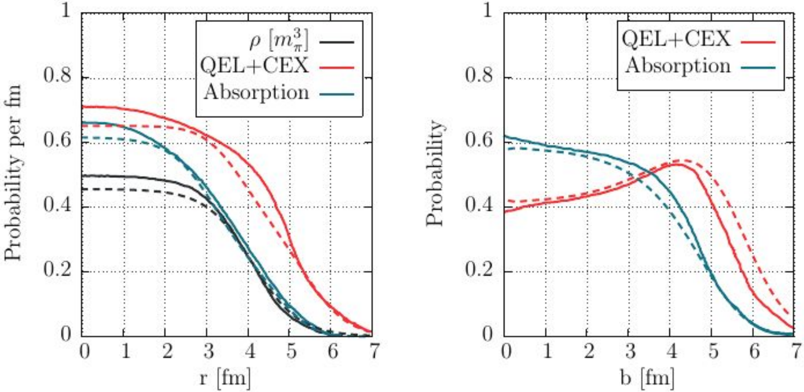
Pion cascade

→ **Oset et al. cross section** model for kinetic energies below 350 MeV

[E. Oset, L.L. Salcedo, D. Strottman, Phys.Lett. B165 (1985) 13–18]

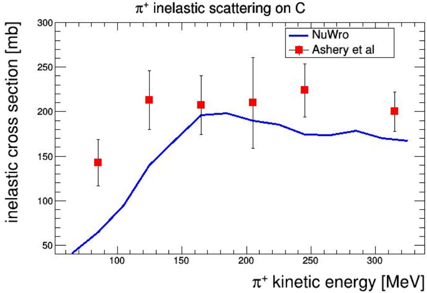
→ **Data driven cross sections** for higher energies

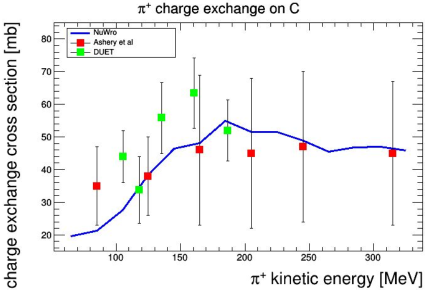
→ **Angular distributions** tuned to **SAID model**

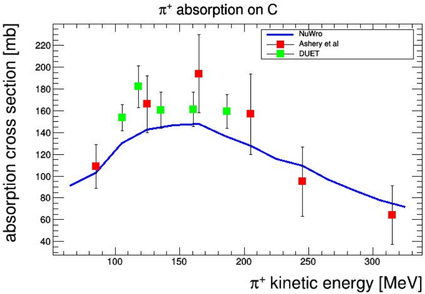


Oset et al. calculations (solid) and NuWro implementation (dashed)

Comparison with π-nucleus scattering data







Δ lifetime and formation time

Pion reinteractions are preceeded by:

$\rightarrow$ Δ propagation for RES

$$t_f = \gamma\tau_\Delta = \gamma\Gamma^{-1}$$

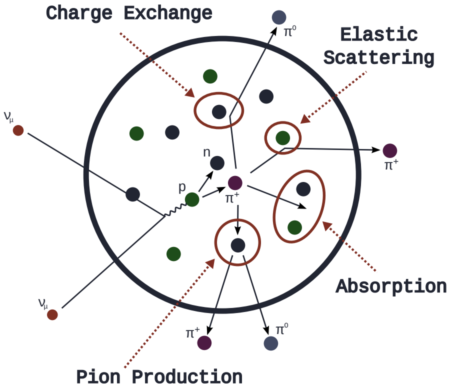
$\rightarrow$ Formation zone for DIS

$$t_f = \tau_0 \frac{EM}{\mu_T^2}$$

E,M - nucleon energy and mass

$\mu_T^2 = M^2 + p_T^2$ - transverse mass

$\tau_0 = 8\ \mathrm{fm}$



T. Golan

Pion production model in NuWro

A simple model with many building blocks:

→ Pion production through the Δ-resonance excitation (Adler)

→ More inelastic processes described by quark-parton model with Pythia6 used for hadronization

→ Hadronization parameters tuned to get a good agreement with experimental data

→ Smooth transition region between "RES" and "DIS"

→ Plane-Wave Impulse Approximation

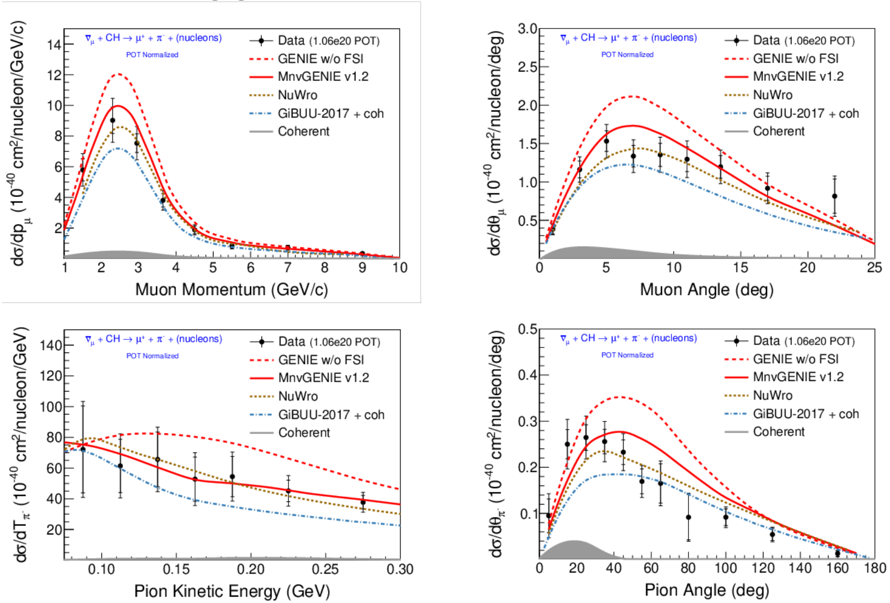
→ Basic nuclear models for nucleon selection (FG, LFG, SF, ...)

→ Δ self-energy effects (Oset, Salcedo)

→ Final state interactions modeled via intranuclear cascade based on Oset et al. model

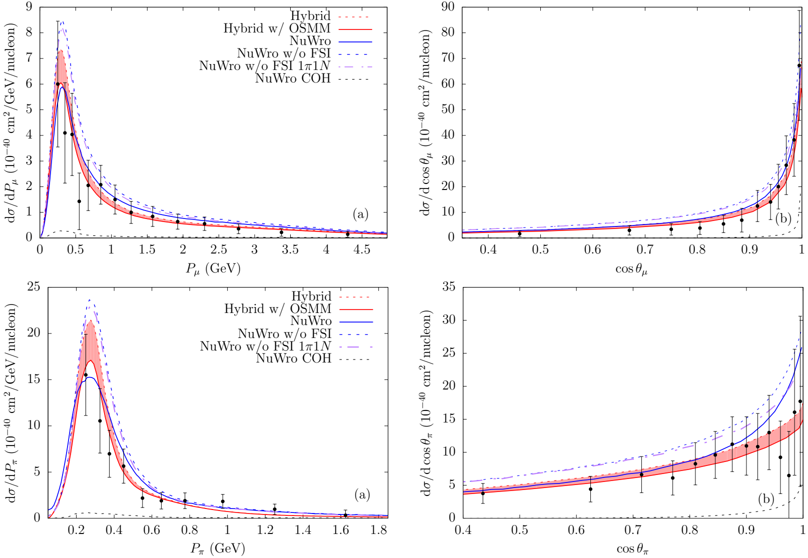
How does this compare to data?

MINERνA: $\bar{\nu}$ CC1π^-



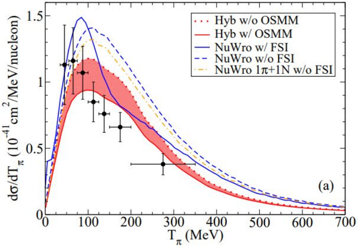
MINERvA Collaboration, Phys.Rev. D100 (2019) 052008

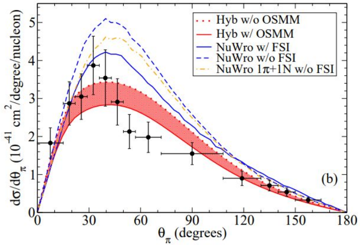
T2K ν CC1π^+



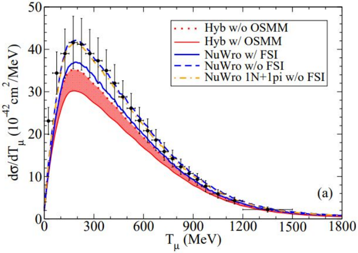
A. Nikolakopoulos, R. González-Jiménez, K. Niewczas, J. Sobczyk, and N. Jachowicz, Phys.Rev. D97 (2018) 093008

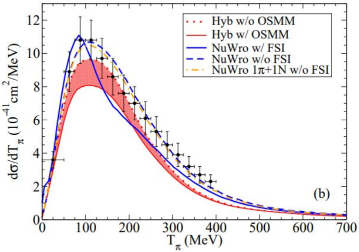
MINERvA ν CC1π^+





MiniBooNE ν CC1π^+





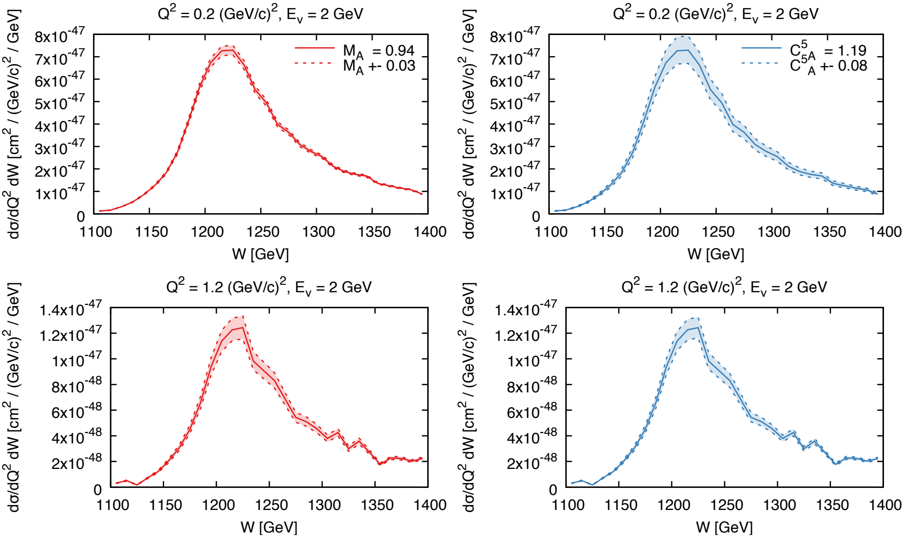
R. González-Jiménez, K. Niewczas, and N. Jachowicz, Phys.Rev. D97 (2018) 013004

Let's run it yourself

NuWro RES parameters

parameter	meaning	value range
delta_FF_set	choice of FFs: **(1)** dipole form of [Phys.Rev. D80 (2009) 093001] (2-5) various options of [Phys.Rev. D71 (2005) 074003] (6) proposed by [Phys.Rev. C57 (1998) 2693–2699] (7) chiral quark model of [Phys.Rev. C75 (2007) 065203]	1 ... 7
pion_axial_mass	M_A if delta_FF_set==1	number (in MeV)
pion_C5A	C_5^A if delta_FF_set==1	number
delta_angular	angular distribution of π from Δ decay: (0) isotropic in Δ rest frame (1) from ANL paper, **(2)** from BNL paper (3) from Rein-Sehgal model	0, 1, 2, 3
spp_precision	precision in the computation of F^{SPP}	integer
res_dis_cut	upper bound in RES/DIS transition	number (in MeV)
bkgrscaling	modification of the non-resonant background	-1.3 ... 1.3

Reweighting



Outlook

→ **NuWro** has a **simple** pion production **model** build with **many independent components**

→ Our **agreement** with **neutrino** pion production **data** is **outstanding**

→ We work **together with** the **Ghent group** to implement more **sophisticated theoretical approaches**

→ We work on **optimizations** of the **event generation procedure** (*talk by A. Nikolakopoulos*)

→ **More incredible results are coming soon!**

NuWro team since 2006

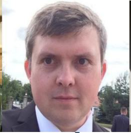
(currently active)

T. Golan

K. Graczyk

C. Juszczak

K. Niewczas

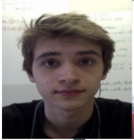


J. Nowak

J.T. Sobczyk

J. Żmuda

new students →



T. Bonus

C. Thorpe

Notable supporters

D. Kiełczewska
(passed away in 2016)

Inspiration

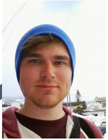
P. Przewłocki

K. Kowalik

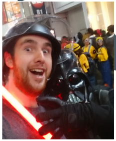
NuWro at T2K

A. Ankowski

Spectral function



L. Pickering



P. Stowell

Reweightning tools

Back-up slides

Dimensionality of the problem

Δ-resonance excitation (free nucleon)	Pion production off a nucleon	Pion production on a nucleus
$\frac{\mathrm{d}^2\sigma}{\mathrm{d}Q^2\mathrm{d}W}$	$\frac{\mathrm{d}^4\sigma}{\mathrm{d}Q^2\mathrm{d}W\mathrm{d}\Omega_\pi^*}$	$\frac{\mathrm{d}^8\sigma}{\mathrm{d}Q^2\mathrm{d}W\mathrm{d}\Omega_\pi^*\mathrm{d}E_m\mathrm{d}\vec{p}_m}$

+2

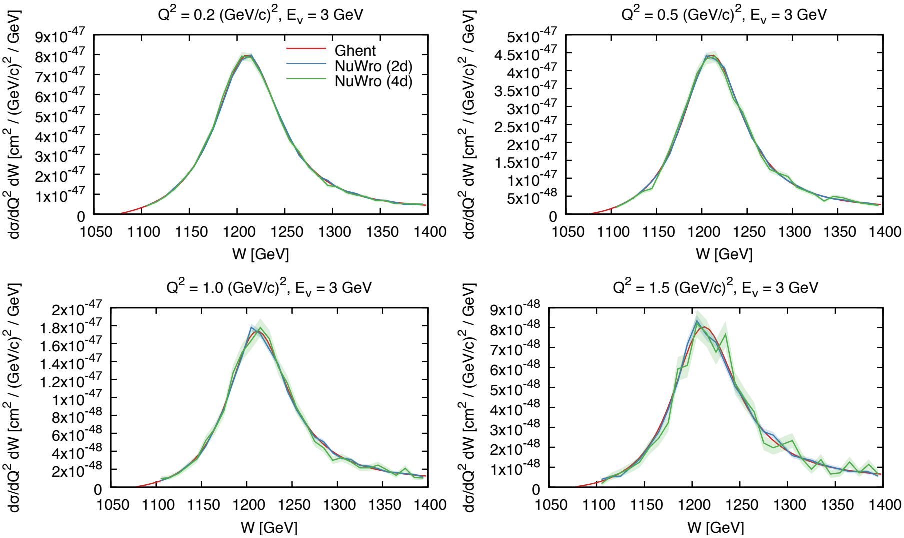
+4

Include angular information about the Δ decay (Ω_π^*)

Put the target nucleon inside a nucleus ($E_m, \vec{p}_m$)

+1 invariant variable: the cross section is always symmetric w.r.t. 1 azimuthal angle, e.g., ϕ_μ

Implementation of the Ghent model



$$\nu_\mu + p \rightarrow \mu^- + p + \pi^+$$